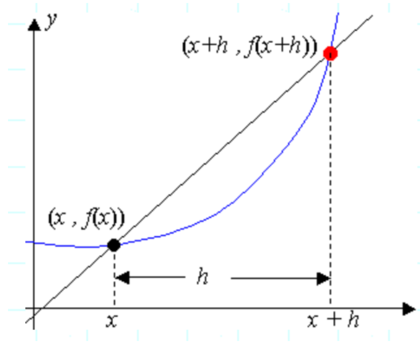
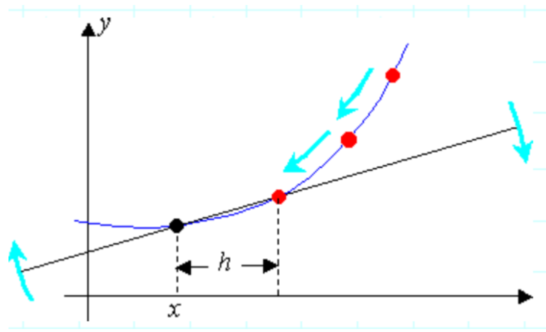
Differentiation from First Principles

- Differentiation gives the gradient function of a curve.
- Differentiation from **first principles** is the long method that works from the basic principle that the gradient of a tangent is $\frac{change\ in\ y}{change\ in\ x}$.





- From the diagram above, $= \frac{y_2-y_1}{x_2-x_1} = \frac{f(x+h)-f(x)}{(x+h)-x} = \frac{f(x+h)-f(x)}{h}$.
- If we let h get so small that it is almost zero, this will give us the gradient function of the curve and thus the gradient of the tangent to the curve at a particular point.

- Therefore, the gradient function of a curve can be found from first principles using the rule: $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$

1) Find the gradient function of $f(x) = x^2 + 5x$ using first principles.

$f(x+h) = (x+h)^2 + 5(x+h) = x^2 + 2xh + h^2 + 5x + 5h$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{(x^2+2xh+h^2+5x+5h)-(x^2+5x)}{h} \\
&= \lim_{h\to 0}\frac{x^2+2xh+h^2+5x+5h-x^2-5x}{h} \\
&= \lim_{h\to 0}\frac{2xh+h^2+5h}{h} \\
&= \lim_{h\to 0}\frac{h(2x+h+5)}{h} \\
&= \lim_{h\to 0} 2x+h+5 \\
&= 2x+5
\end{aligned}
$$

2) Find the gradient function of $f(x) = 2x^3$ using first principles.

$f(x+h) = 2(x+h)^3 = 2(x+h)(x+h)(x+h) = 2x^3 + 6x^2h + 6h^2x + 2h^3$

$$
\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \\
&= \lim_{h\to 0}\frac{(2x^3+6x^2h+6h^2x+2h^3)-(2x^3)}{h} \\
&= \lim_{h\to 0}\frac{2x^3+6x^2h+6h^2x+2h^3-2x^3}{h} \\
&= \lim_{h\to 0}\frac{6x^2h+6h^2x+2h^3}{h} \\
&= \lim_{h\to 0}\frac{h(6x^2+6hx+2h^2)}{h} \\
&= \lim_{h\to 0} 6x^2+6hx+2h^2 \\
&= 6x^2
\end{aligned}
$$

Delta Ex 5.7 pg 77, Q1 – 10
Extension Q11 - 13